



Progressive Education Society's  
Modern College of Arts, Science and Commerce Ganeshkhind, Pune-16  
(Autonomous)  
Internal Examination: Jan.2022  
Faculty: Science and Technology

Program: BSc Comp05  
Program: BSC(Computer Science)  
Class: FYBSc(Comp.Sc.)  
Name of the Course: Matrix Algebra  
Course Code: MTC-111  
Paper: I

Semester: I

SET: A  
Course: CORE  
Max. Marks: 35  
Time: 2Hr

**Instructions to Candidates:** Instructions to the candidate:

- 1) There are 4 sections in the question paper. Write each section on separate page.
- 2) All Sections are compulsory.
- 3) Figures to the right indicate full marks.
- 4) Draw a well labelled diagram wherever necessary

**SECTION: A**

**1 Choose the correct option (5M)**

1) If Matrix  $A$  is of order  $3 \times 5$  and Matrix  $B$  is of order  $5 \times 2$ , then order of  $AB$  is:

- a)  $3 \times 3$               b)  $3 \times 2$   
c)  $3 \times 5$               d)  $5 \times 5$

2) The following matrix of a system has :

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 \\ 0 & 3 & 4 & | & 2 \\ 0 & 0 & 2 & | & 2 \end{bmatrix}$$

- a) Trivial solution              b) infinitely many Solution  
c) No solution                  d) unique solution

3) Which of the following augmented matrix correspond to the given system

$$a + 2b = 3$$

$$b + 3c = 6$$

$$b + c = 4$$

a)  $\begin{bmatrix} 1 & 2 & 0 & | & 3 \\ 0 & 1 & 3 & | & 6 \\ 0 & 1 & 1 & | & 4 \end{bmatrix}$

b)  $\begin{bmatrix} 1 & 2 & 0 & | & 3 \\ 1 & 3 & 0 & | & 6 \\ 1 & 1 & 0 & | & 4 \end{bmatrix}$

$$\text{c) } \left[ \begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 2 & 1 & 1 & 6 \\ 0 & 3 & 1 & 4 \end{array} \right] \quad \text{d) } \left[ \begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 1 & 3 & 0 & 6 \\ 0 & 1 & 1 & 4 \end{array} \right]$$

4) The set of all linear combinations of the columns of  $A$  is known as:

- a) Column space of  $A$       b) Null  $A$   
c) Row Space of  $A$       d) Null Space of  $A^T$

5) If  $\det(A) = 5$ , then,  $\det(A^T) = ?$

- a) 2      b) -1  
c) 5      d) 0

## 2 Very short answer questions ( any 4/6) (4M)

1) If  $A = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$  and  $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ , find  $A - 2B + 3I$

2) Use the definition of  $AX$  and write the following vector equation as a matrix equation:

$$z_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} + z_2 \begin{bmatrix} 2 \\ 3 \end{bmatrix} + z_3 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + z_4 \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$$

3) Define linearly independent set

4) Determine whether  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is a linear transformation or not:

$$T(x, y) = (x^2, y^2)$$

5) Let,  $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ , then find  $\det A$  and then using property of determinants find  $\det(5A)$

6) Give the standard matrix of reflection through  $y = -x$  line.

## SECTION:B

## 3 Short answer questions ( any 4/6) (8M)

1) Let  $A = \begin{bmatrix} 6 & 1 \\ -2 & 3 \end{bmatrix}$  and  $B = \begin{bmatrix} 5 & -2 \\ 4 & 4 \end{bmatrix}$  Verify  $(A + B)^T = A^T + B^T$

2) Find the reduced row echelon form of the matrix  $A = \begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$

3) Let  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  be linear transformation such that  $T(X) = AX$  find images of  $u = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$   $v = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$  where

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

4) solve the following system of linear equations.

$$\begin{aligned} x - y &= 3 \\ 5x + y &= 2 \end{aligned}$$

5) If  $T$  is linear transformation then show that,

i)  $T(c.u + d.v) = c.T(u) + d.T(v)$  where  $c$  and  $d$  are scalars

ii)  $T(0) = 0$ .

6) Find the determinant of  $A = \begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$ .

## SECTION:C

### 4 Short answer questions (Attempt any 4/6) (8M)

1) Let  $A = \begin{bmatrix} 1 & 1 \\ 3 & -2 \end{bmatrix}$  and  $B = \begin{bmatrix} 2 & 1 \\ 4 & 0 \end{bmatrix}$  be two matrices show that matrix multiplication is not commutative, i.e.  $AB \neq BA$ .

2) Determine whether matrix  $A$  is invertible or not. If yes, find its inverse.  $A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}$

3) Find the general solution of the linear system whose augmented matrix is

$$[A|B] = \left[ \begin{array}{ccc|c} 1 & -4 & -5 & 0 \\ 0 & 1 & -1 & 1 \end{array} \right]$$

.

4) Find the parametric equation of the line through  $a$  parallel to  $b$  where

$$a = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

5) Define Subspace.

Check whether  $H = \{(x, y) / y = 3x + 2\}$  is a subspace of  $\mathbb{R}^2$  or not ?

6) State Dimension and Rank Theorem.

Let  $[A]_{3 \times 5}$  be matrix having rank 2 then what will be the nullity of matrix  $A$ ?

SECTION:D

**5 Attempt any two of the following (2/4) (10M)**

1) Use Cramer's rule to compute the solution of the following system.

$$\begin{aligned}x + y + 2z &= 7 \\ -x - 2y + 3z &= 6 \\ 3x - 7y + 6z &= 1\end{aligned}$$

2) Let  $A = \begin{bmatrix} -3 & 0 & 6 \\ -2 & 2 & 3 \\ 0 & -6 & 3 \end{bmatrix}$  and  $p = \begin{bmatrix} 1 \\ 14 \\ -9 \end{bmatrix}$  Determine if vector  $p$  is in  $\text{Col}A$  ?

3) Determine if the following vectors are linearly independent or not?

$$\begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -4 \end{bmatrix}$$

4) Determine if vector  $b$  is linear combination of vectors  $a_1, a_2$  and  $a_3$ .

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \quad a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \quad a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} \quad b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$